Chapter 8
Bearing Capacity of Floating Ice

8-1. Introduction

When a river, lake, or sea is subjected to air temperatures below the freezing point for an extended period of time, an ice cover forms, whose thickness depends on the intensity of this freezing temperature, its duration, and other factors. This subject is discussed in detail in Chapter 2 of this manual. The thickness of this ice cover may be measured mechanically by a crew that operates on the ice cover. It may also be measured remotely, say from an airplane or balloon, by means of electromagnetic waves. This chapter discusses the bearing capacity of floating ice and the methods of predicting it.

8-2. Bearing Capacity of Ice Blocks

First, consider an ice block of uniform thickness floating in water (Figure 8-1). Because the specific weight of ice is less than that of water, the ice block floats. It may also carry an additional load *P*.

a. To determine the bearing capacity of this floating block, subjected centrally to a static load *P*, consider its vertical equilibrium before it is totally submerged, as shown in Figure 8-1. The equilibrium equation, for $z \leq h$, is

$$P + Ah\,\gamma_i = Az\,\gamma_w \tag{8-1}$$

where

P = vertical resultant of the load

A = horizontal area of the ice block

γ_w = specific weight of water

γ_i = specific weight of ice.

The other symbols are defined in Figure 8-1. Bearing capacity is reached when z approaches h. Substituting the limiting case $h = z$ into Equation 8-1 yields

$$P_{max} = Ah\,(\gamma_w - \gamma_i). \tag{8-2}$$

This is the largest load the ice block can carry. For larger loads *P*, the block will sink. It may be of interest to note that without the load *P*, the submerged depth of the block, according to Equation 8-1, is

$$z_o = h\,\frac{\gamma_i}{\gamma_w}. \tag{8-3}$$

b. Following are two illustrative examples of Equation 8-2.

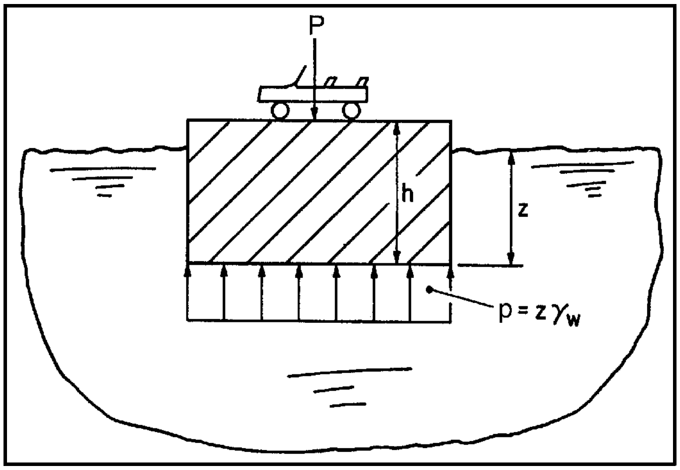

Figure 8-1. Floating ice block with a centrally placed load *P*

(1) Determine the bearing capacity of an ice block of thickness h = 3 feet (0.9 meters) and a surface area A = 100 ft^2 (9.29 m^2) for a centrally placed static load P. According to Equation 8-2, with γ_w = 62.4 lb/ft^3 (1000 kg/m^3) and γ_i = 57.3 lb/ft^3 (918 kg/m^3), the largest weight the ice block can carry is

$$P_{max} = 100 \times 3 \times (62.4 - 57.3) = 1530 \text{ lb.}$$

In SI units

$$P_{max} = 9.29 \times 0.9 \times (1000 - 918) = 686 \text{ kg.}$$

(2) For an ice block of constant thickness h = 2 feet (0.6 meters), determine the surface area A needed to carry a centrally placed load of P = 3000 pounds (1361 kilograms). From Equation 8-2 it follows that the required area is

$$A_{req} \geq \frac{P}{h\left(\gamma_w - \gamma_i\right)} .$$

With γ_w = 62.4 lb/ft^3 (1000 kg/m^3) and γ_i = 57.3 lb/ft^3 (918 kg/m^3), it follows that

$$A_{req} \geq \frac{3000}{2\left(62.4 - 57.3\right)} = 294 \text{ ft}^2 \text{ (or 27.7 m}^2 \text{ in SI units).}$$

For example, a square area of 17 × 17.3 (5.2 × 5.3 meters) will achieve this aim.

c. When the resultant of the load is not centrally placed on the ice block, the ice block will tilt. This will result in a linearly varying pressure p at the bottom surface of the block. The bearing capacity for this case may be determined as done previously, except that now, in addition to vertical equilibrium, the moment equilibrium has to be considered. Note that when the eccentricity of the load resultant exceeds a certain limit, namely when the loading moment is larger than the restoring moment, the ice block will tip over. When the load is dynamic the analysis is more involved. Then, the equations of motion for the ice block have to be coupled with the dynamic equations for the fluid base.

8-3. Bearing Capacity of Ice Covers for Loads of Short Duration

When the dimensions of the ice plate are very large compared to its thickness, the ice cover is relatively flexible in the vertical direction and a vertical load *P* will deflect the plate, as shown in Figure 8-2. Thus, in addition to the constant pressure p_o caused by the uniform weight of the ice cover, there will also occur a variable pressure $p(x,y)$ caused by the boat effect of the deflected cover (Ashton 1986).

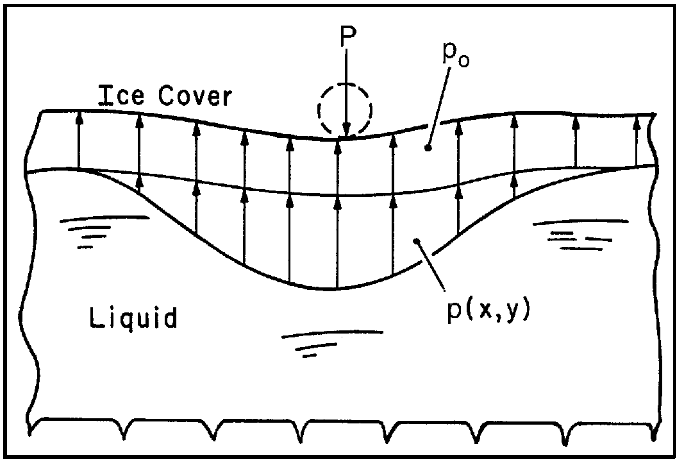

Figure 8-2. Deflection of large ice plate from vertical load *P*

a. As is well known, sufficiently large loads will break through the ice. For this problem, the bearing capacity of the ice cover depends on the strength of the cover to resist the vertical deformations.

b. To date there is no generally accepted method for calculating the bearing capacity of flexible ice covers. One of the reasons is that ice is a complex material, complicated further by the fact that the bottom part of a floating cover is near the melting temperature. Another reason is a lack of researcher interest in break-through tests on frozen lakes or rivers, especially at low temperatures. Still another is the general lack of coordination between the theoretical and empirical testing efforts conducted throughout the world. In the meantime, there is a need for estimating the bearing capacity of large ice covers subjected to a variety of loads. To achieve this, a number of approaches used in various countries are presented and discussed.

8-4. Experience Values

Individuals who often use ice covers for transportation develop a knowledge of the bearing capacity of ice covers of given thickness and quality through experience. To enable an interested party to make a rough estimate of the ice thickness needed for safe movement of people and vehicles, Table 8-1 is included. In determining the effective ice thickness to be used with Table 8-1, any thickness of “snow” ice (i.e., ice that is white owing to entrained air bubbles) should be set as equivalent to half that thickness of clear “black” ice. For example, if the measured thickness of the ice cover is 76.2 centimeters (30 inches), of which 25.4 centimeters (10 inches) is snow ice, the effective ice thickness should be considered as 50.8 + 25.4/2 = 63.5 centimeters (20 + 10/2 = 25 inches).

Table 8-1
Approximate Ice Load-Carrying Capacity (Note: Read the text before using table.)

Type of Vehicle	Total Weight†, Metric tons (tons)	Necessary Ice Thickness* at Average Ambient Temperature for Three Days - cm (inches): 0 to -7 °C (32 to 20 °F)	-9 °C and Lower (15°F and Lower)	Distance Between Vehicles m (ft)
Tracked	6.6 (6)	25.4 (10)	22.9 (9)	15.2 (50)
	11.0 (10)	30.5 (12)	27.9 (11)	19.8 (65)
	17.6 (16)	40.6 (16)	35.6 (14)	24.4 (80)
	22.0 (20)	45.7 (18)	40.6 (16)	24.4 (80)
	27.6 (25)	50.8 (20)	45.7 (18)	30.5 (100)
	33.1 (30)	55.9 (22)	48.3 (19)	35.1 (115)
	44.1 (40)	63.5 (25)	55.9 (22)	39.6 (130)
	55.1 (50)	68.6 (27)	63.5 (25)	39.6 (130)
	66.1 (60)	76.2 (30)	71.1 (28)	45.7 (150)
Wheeled	2.2 (2)	17.8 (7)	17.8 (7)	15.2 (50)
	4.4 (4)	22.9 (9)	20.3 (8)	15.2 (50)
	6.6 (6)	30.5 (12)	27.9 (11)	19.8 (65)
	8.8 (8)	33.0 (13)	30.5 (12)	32.0 (105)
	11.0 (10)	38.1 (15)	35.6 (14)	35.1 (115)

* Freshwater ice.
† When the temperature has been 0 °C (32 °F) or higher for a few days, the ice is probably unsafe for any load.

8-5. Empirical Methods

An often used formula for single vehicles is

$$P = Ah^2 \tag{8-4}$$

where P is the allowable load, h is the effective ice cover thickness, and A is a coefficient that depends on the quality of the ice, the ice temperature, the geometry of the load, the kind of units used, and the factor of safety. To ensure safe movement of single vehicles crossing lake or river ice at temperatures below 0°C (32°F), the straightforward and practical formulas $P = h^2/16$ or $h = 4\sqrt{P}$ have been used for decades. These formulas are for English units in which P is in tons and h is in inches. Although not strictly equivalent, similar practical formulas for SI units are $P = h^2/100$ or $h = 10\sqrt{P}$, where P is in metric tons (1000 kg) and h is in centimeters, and $P = h^2$ or $h = \sqrt{P}$, where P is in meganewtons and h is in meters. These formulas are all for black ice below 0°C (32°F), and appropriate adjustments to thicknesses to account for snow ice should be computed as in paragraph 8-4. The following are illustrative examples of Equation 8-4.

a. Determine the allowable load of an ice cover with the smallest ice thickness h = 25.4 centimeters (10 inches).

$$P = \frac{h^2}{16} = \frac{10 \times 10}{16} = 6.25\, tons.$$

In metric units, this is

$$P = \frac{h^2}{100} = \frac{25.4 \times 25.4}{100} = 6.45 \times 10^3 \text{ kilograms (6.45 metric tons)}.$$

b. Determine the smallest ice thickness needed to safely carry one person of weight P = 200 lb = 0.1 ton (90.7 kilograms = 0.0907 metric ton).

$$h = 4\sqrt{P} = 4\sqrt{0.1} = 1.26 \text{ inches}$$

Expressed in metric units, the required thickness is

$$h = 10\sqrt{P} = 10\sqrt{0.0907} = 3.0 \text{ centimeters}.$$

8-6. Method Based on the Theory of Elastic Plates

An analytical method for determining the bearing capacity of an ice cover for loads of short duration is based on the elastic bending theory of thin plates in conjunction with a crack criterion. The method consists of the following three steps

- Determination of the maximum stress σ_{max} in the floating plate attributable to a given load.

- Determination of the load P_{cr} at which the first crack occurs, utilizing the crack criterion $\sigma_{max} \leq \sigma_f$, where σ_f is the failure stress.

- Correlation of P_{cr} with the breakthrough load P_f. This step is needed because, according to field tests for various plate and load geometries, the occurrence of the first crack does not cause breakthrough.

The failure stress σ_f is usually obtained by loading a floating ice beam, cut out from the ice cover under consideration as shown in Figure 8-3, to failure and then by computing the largest bending stress at which it failed.

a. With the equation $D\nabla^4 w + \gamma w = q$ for the response of a homogeneous ice cover, in which w is its deflection, subjected to a uniform load q over a circular area, as shown in Figure 8-4, in conjunction with the crack criterion $\sigma_{max} < \sigma_f$,

$$P_{cr} = \left(\frac{1}{3(1 + \nu)C(\alpha)} \right) \sigma_f h^2 \tag{8-5}$$

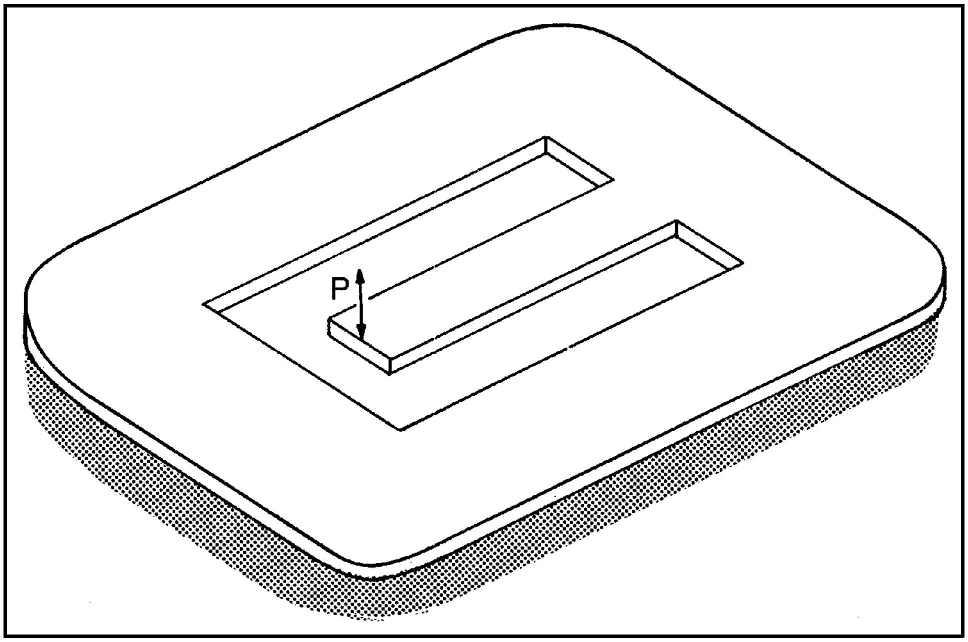

Figure 8-3. Stress test of floating ice beam

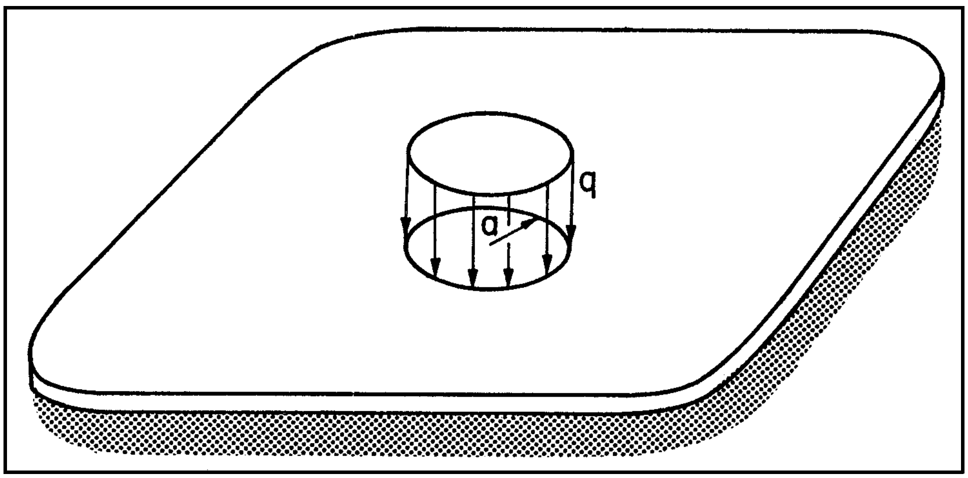

Figure 8-4. Homogeneous ice cover subjected to a uniform load over a circular area

where

h = ice cover thickness

v = Poisson's ratio of the ice cover

a = radius of the loaded area subjected to the uniform load

$q = P/(\pi a^2)$

$\alpha = a/(D/\gamma)^{1/4}$

γ = specific weight of the liquid base

$D = Eh^3/[12(1 - v^2)]$

E = Young's modulus of the ice cover

∇^4 = biharmonic operator, e.g. $(\partial^4/\partial x^4 + 2\partial^4/\partial x^2\partial y^2 + \partial^4/\partial y^4)$ in Cartesian coordinates

$C(\alpha)$ = given in Figure 8-5.

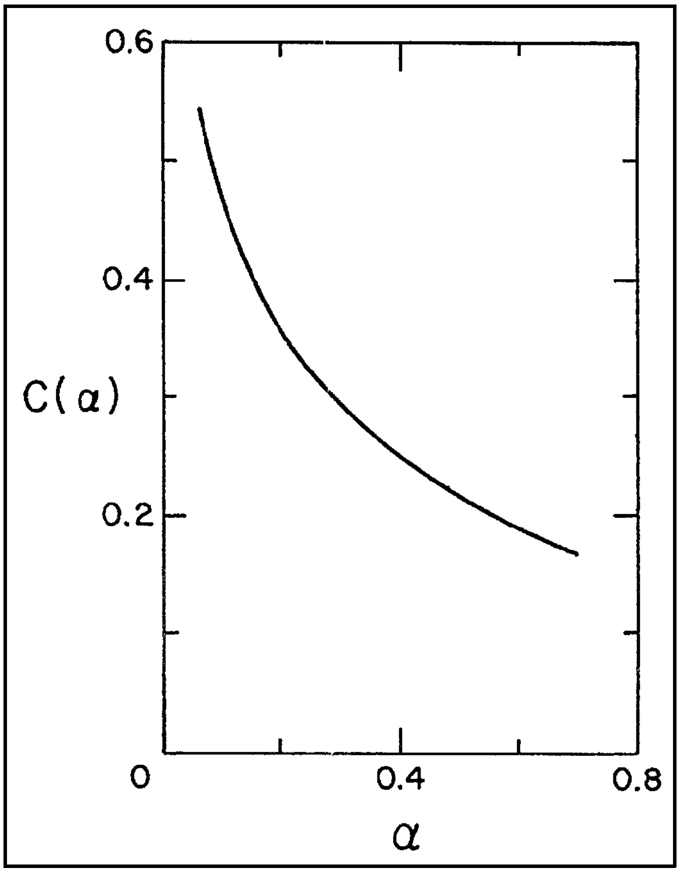

Figure 8-5. *C*(α) for Equation 8-5

b. To demonstrate the use of Equation 8-5, consider an ice cover with h = 30.5 centimeters (12 inches), E = 3.45 GPa (500,000 lbf/in.^{2}), v = 0.34, for a circular load distribution with radius a = 102 centimeters (40 inches). According to Figure 8-6, the resulting $(D/\gamma)^{1/4}$ = 554 centimeters (218 inches) and hence α = 40/218 (or 102/554) = 0.18. According to Figure 8-5, the corresponding $C(\alpha)$ = 0.38. Thus

$$P_{cr} = \left(\frac{1}{3(1 + 0.34)0.38} \right) \sigma_f h^2 = 0.65 \, \sigma_f h^2 .$$

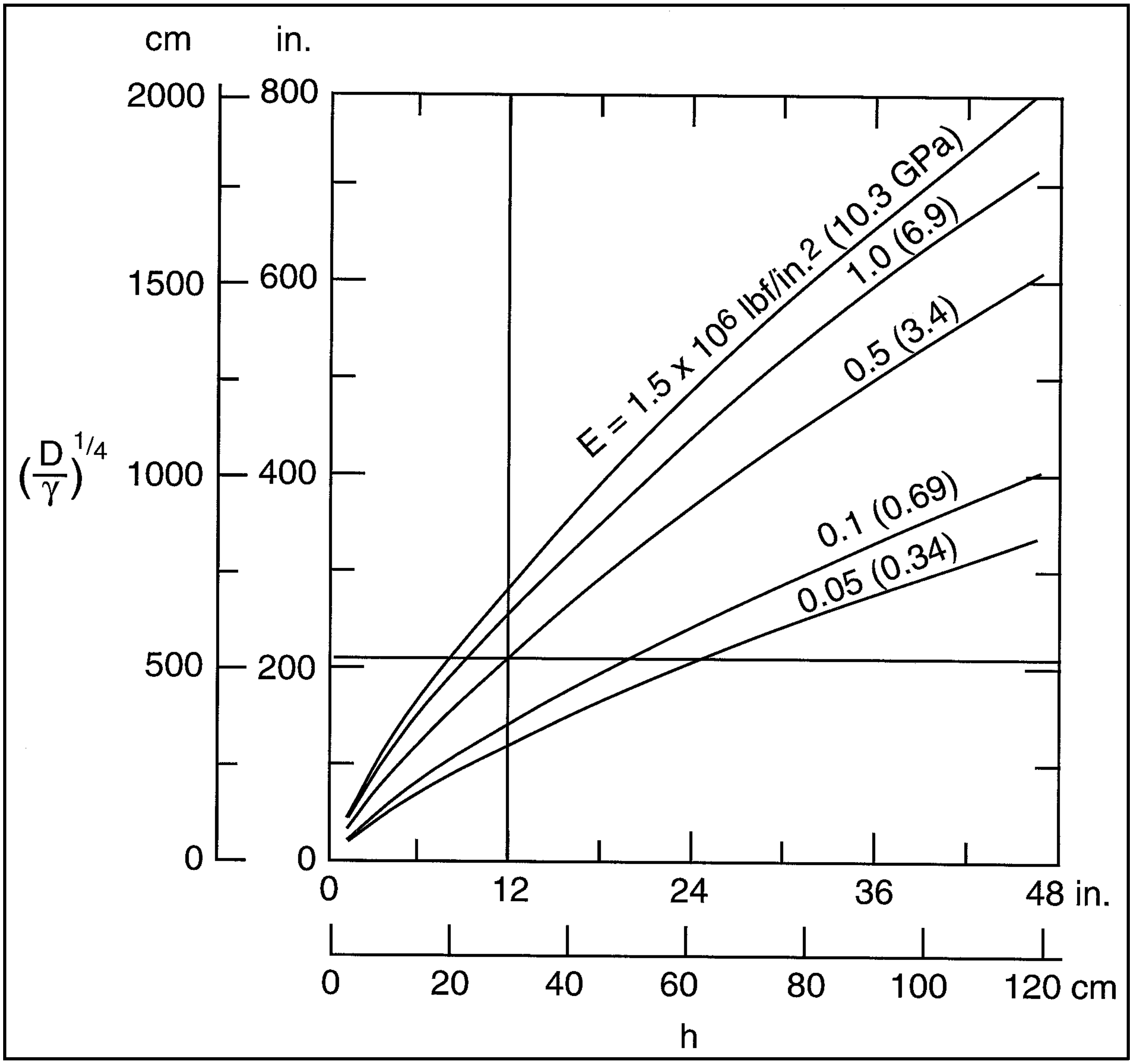

Figure 8-6. Graph required to solve Equation 8-5

8-7. Bearing Capacity of Ice Covers for Loads of Long Duration

For loads that do not cause an instantaneous breakthrough, the ice cover deforms at first elastically, and then with time it continues to deform by creep, especially in the vicinity of the load. Depending upon the load intensity and geometry, as well as upon the ice cover properties, the resulting time displacement graph may be of the type shown in Figure 8-7. In the case of the upper curve (I), although the ice cover was able to carry the load immediately after the load was placed on it, there exists a "time to failure" t_f at which the load breaks through the cover. Attempts to analyze problems of this type have not been conclusive to date. In the absence of a reliable method for predicting the bearing capacity of ice plates subjected to loads of long duration (storage of equipment, parking of vehicles, and airplanes), Figure 8-8 is presented for estimates. Note the drop of the breakthrough load with time. Thus, a stored item that is safe when placed on the ice cover may break through after a certain time period t_f.

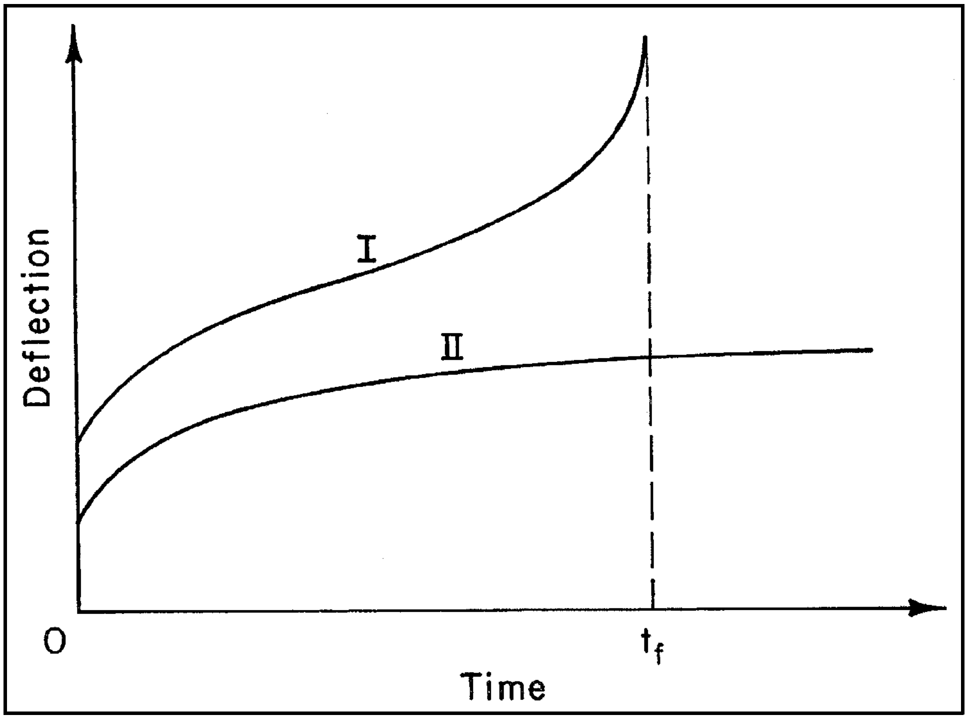

Figure 8-7. Time-displacement graph of ice deformity

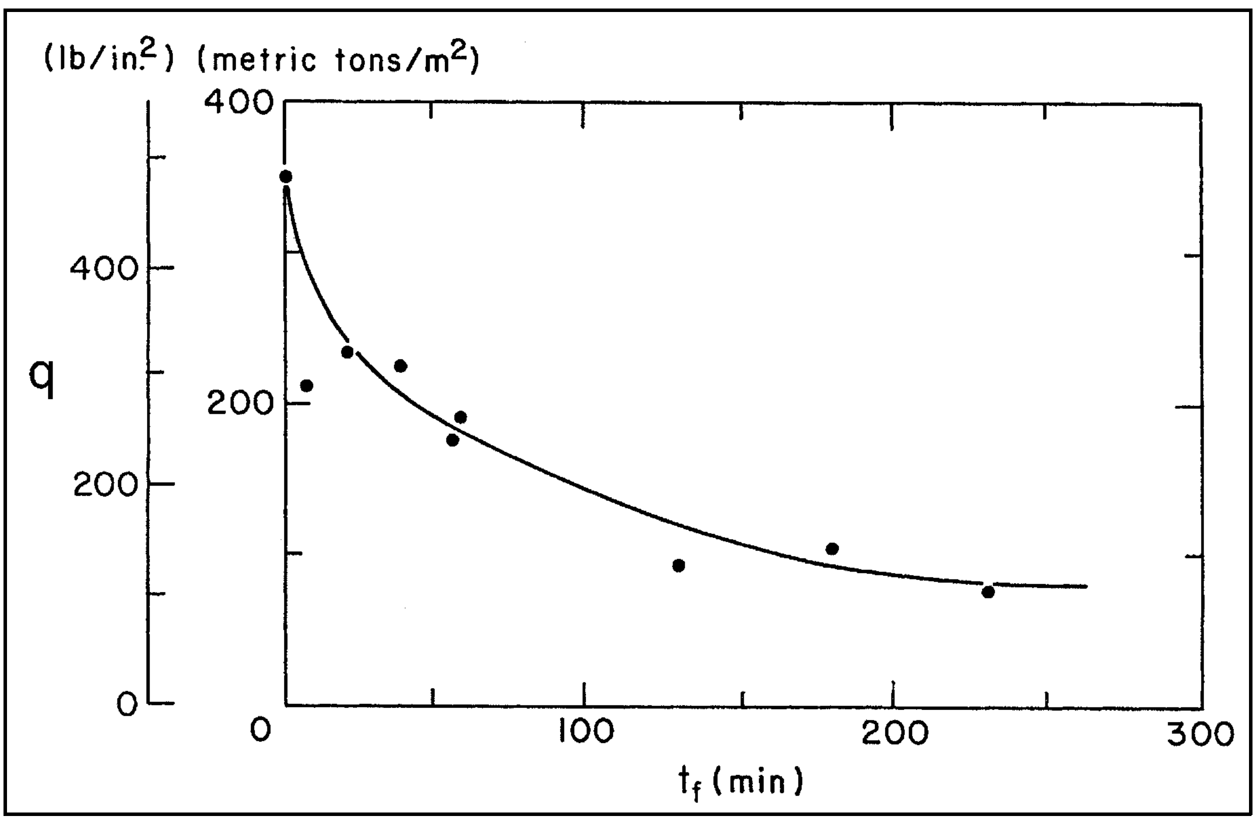

Figure 8-8. Bearing capacity reduction for loads of long duration

8-8. Other Considerations

Practically, there are a few things one should be aware of when operating on the ice. Cracks are almost always present because of thermal expansion. Most of these are called "dry cracks" because they do not go to the bottom of the ice sheet (note the concept that all thermal expansion is in the upper portion of the sheet because the bottom is always at 0°C [32°F]). These dry cracks do not have an appreciable effect on bearing capacity. However, wet cracks that do penetrate the entire sheet should be approached at 50 percent acceptable load and one should try to cross them at an angle near 90 degrees.

a. When parked, i.e., the long-term load situation, one should look for any signs of water coming up through the ice and beginning to flood the area. If this occurs, *MOVE*, because breakthrough is almost inevitable; this water is an additional load.

b. Throughout the preceding paragraphs, we have stressed the importance of temperature and inferred daily average air temperatures. This was done because this is the information available, but it is the ice temperature that is important. So, remember that snow cover, as an insulation, slows ice temperature change. Also, experience has shown that an added safety factor is necessary when there has been a recent big drop in air temperature. Apparently, this causes a thermal shock leading to additional cracking. Many accidents to experienced operators have occurred after a rapid drop in air temperature.

8-9. References

a. *Required publications.*
None.

b. *Related publications.*

Ashton 1986
Ashton, G.D., ed. 1986. *River and Lake Ice Engineering*, Water Resources Publications, Littleton, Colorado.